# Math Competition Preparation... #1

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1. **Rules & Interesting Techniques**

   (a) In contest problems, one often needs to square a number very quickly. There are many ways of quickly squaring a number, the method presented below was shown to me by David Linker of NYC Math Team / CCNY.

   First, observe that the following equality holds:

   $$a^2 = (a+b)(a-b) + b^2$$

   Given the above equality, it is possible to evaluate $a^2$ quickly if $b$ is selected properly (Hint: usually by making $(a+b)$ or $(a-b)$ a number ending with 0). See below for examples:

   $$\begin{aligned} 19^2 &= (19+1)(19-1) + 1^2 \\ 19^2 &= (20)(18) + 1 \\ 19^2 &= 361 \end{aligned}$$

   (b) A quadratic is an equation of the form $ax^2 + bx + c = 0$. The equation has at most two solutions, of the form $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Observe the following equal arithmetic expressions:

   $$\begin{aligned} &(x-a)(x-b) \\ &x^2 - ax - bx + ab \\ &x^2 - (a+b)x + ab \end{aligned}$$

   From the above relations, if an expression $ax^2 + bx + c$ has roots $p, q$. Then the sum of the roots $(p+q) = \frac{-b}{a}$, and the product of the roots $pq = \frac{c}{a}$.

2. **Problems**

(a) Evaluate the squares of all integers from 80 to 99 mentally.

(b) Find a polynomial equation in standard form with integer coefficients with roots: $\frac{-4}{5}, \frac{2}{7}$

(c) Compute $b$ given $x^2 + bx + 6 = 0$ and that one root is twice the other

(d) If $q$ and $r$ are the roots of $x^2 - 5x + 7 = 0$, find $q^2 + r^2$

(e) A man born in the eighteenth century was $x$ years old in the year $x^2$. How old was he in 1776? (Make no correction for calendric changes.)

(f) If $x + \frac{1}{x} = 3$. Evaluate the following expressions:

i. $x^2 + \frac{1}{x^2}$

ii. $x^3 + \frac{1}{x^3}$

iii. $x^4 + \frac{1}{x^4}$

(g) If $q$ and $r$ are the roots of $x^2 + 2x + 5 = 0$. Find the equation whose roots are:

i. $q^2, r^2$

ii. $q^3, r^3$

iii. $\frac{1}{q}, \frac{1}{r}$

iv. $q + 1, r + 1$